Edexcel Maths FP1

Topic Questions from Papers

Coordinates

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3. The rectangular hyperbola, H, has parametric equations $x = 5t$, $y = \frac{5}{t}$, $t \neq 0$.

(a) Write the cartesian equation of H in the form $xy = c^2$.

(1)

Points A and B on the hyperbola have parameters $t = 1$ and $t = 5$ respectively.

(b) Find the coordinates of the mid-point of AB.

(3)

Leave blank

8. A parabola has equation $y^2 = 4ax$, $a > 0$. The point Q $(aq^2, 2aq)$ lies on the parabola.

(a) Show that an equation of the tangent to the parabola at Q is

$$yq = x + aq^2.$$

(4)

This tangent meets the y-axis at the point R.

(b) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q.

(3)

(c) Show that l passes through the focus of the parabola.

(1)

(d) Find the coordinates of the point where l meets the directrix of the parabola.

(2)

Leave blank

6. The parabola C has equation $y^2 = 16x$.

(a) Verify that the point $P(4t^2, 8t)$ is a general point on C.
(1)

(b) Write down the coordinates of the focus S of C.
(1)

(c) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3$$
(5)

The normal to C at P meets the x-axis at the point N.

(d) Find the area of triangle PSN in terms of t, giving your answer in its simplest form.
(4)

Leave blank

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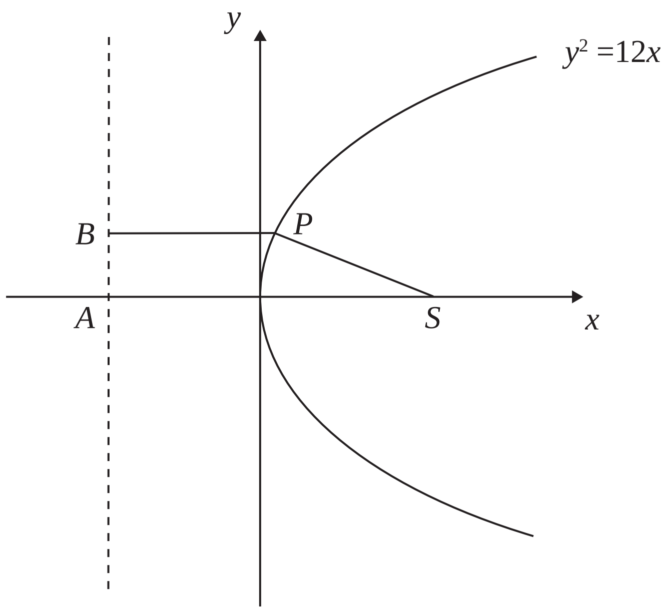

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 12x$.

The point P on the parabola has x-coordinate $\frac{1}{3}$.

The point S is the focus of the parabola.

(a) Write down the coordinates of S.

(1)

The points A and B lie on the directrix of the parabola.
The point A is on the x-axis and the y-coordinate of B is positive.

Given that $ABPS$ is a trapezium,

(b) calculate the perimeter of $ABPS$.

(5)

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7. The rectangular hyperbola H has equation $xy = c^2$, where c is a constant.

The point $P\left(ct, \frac{c}{t}\right)$ is a general point on H.

(a) Show that the tangent to H at P has equation

$$t^2 y + x = 2ct$$

(4)

The tangents to H at the points A and B meet at the point $(15c, -c)$.

(b) Find, in terms of c, the coordinates of A and B.

(5)

Leave blank

5. The parabola C has equation $y^2 = 20x$.

(a) Verify that the point $P(5t^2, 10t)$ is a general point on C.

(1)

The point A on C has parameter $t = 4$.
The line l passes through A and also passes through the focus of C.

(b) Find the gradient of l.

(4)

Leave blank

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point A on H has x-coordinate $3c$.

(a) Write down the y-coordinate of A.
(1)

(b) Show that an equation of the normal to H at A is

$$3y = 27x - 80c$$
(5)

The normal to H at A meets H again at the point B.

(c) Find, in terms of c, the coordinates of B.
(5)

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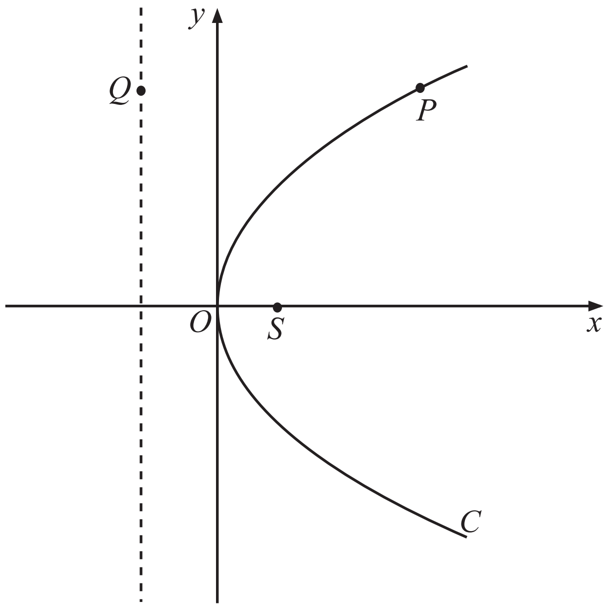

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 36x$.
The point S is the focus of C.

(a) Find the coordinates of S.

(1)

(b) Write down the equation of the directrix of C.

(1)

Figure 1 shows the point P which lies on C, where $y > 0$, and the point Q which lies on the directrix of C. The line segment QP is parallel to the x-axis.

Given that the distance PS is 25,

(c) write down the distance QP,

(1)

(d) find the coordinates of P,

(3)

(e) find the area of the trapezium $OSPQ$.

(2)

Leave blank

10. The point $P\left(6t, \frac{6}{t}\right), t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$.

(a) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t}$$

(5)

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(b) Find the coordinates of A and B.

(7)

Leave blank

8. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C.

(a) Find the equation of the directrix of C.

(2)

(b) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0$$

(4)

The tangent to C at the point $(3, 12)$ meets the directrix of C at the point X.

(c) Find the coordinates of X.

(4)

Leave blank

3. A parabola C has cartesian equation $y^2 = 16x$. The point $P(4t^2, 8t)$ is a general point on C.

(a) Write down the coordinates of the focus F and the equation of the directrix of C.
(3)

(b) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.
(5)

Leave blank

9. The rectangular hyperbola H has cartesian equation $xy = 9$

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on H, where $p \neq \pm q$.

(a) Show that the equation of the tangent at P is $x + p^2 y = 6p$.

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at the point P and the tangent at the point Q intersect at R.

(c) Find, as single fractions in their simplest form, the coordinates of R in terms of p and q.

(4)

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5.

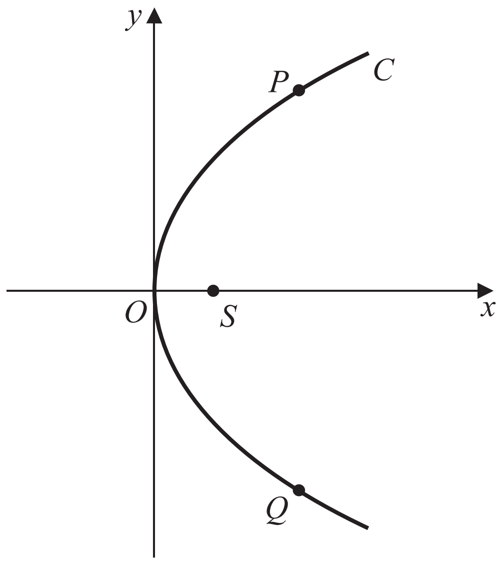

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
The point P lies on C, where $y > 0$, and the point Q lies on C, where $y < 0$
The line segment PQ is parallel to the y-axis.

Given that the distance PQ is 12,

(a) write down the y-coordinate of P,

(1)

(b) find the x-coordinate of P.

(2)

Figure 1 shows the point S which is the focus of C.
The line l passes through the point P and the point S.

(c) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that an equation for the tangent to H at P is

$$x + t^2y = 2ct$$

(4)

The tangent to H at the point P meets the x-axis at the point A and the y-axis at the point B.

Given that the area of the triangle OAB, where O is the origin, is 36,

(b) find the exact value of c, expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

Leave blank

7. The rectangular hyperbola, H, has cartesian equation $xy = 25$

The point $P\left(5p, \frac{5}{p}\right)$, and the point $Q\left(5q, \frac{5}{q}\right)$, where $p, q \neq 0, p \neq q$, are points on the rectangular hyperbola H.

(a) Show that the equation of the tangent at point P is

$$p^2y + x = 10p$$

(4)

(b) Write down the equation of the tangent at point Q.

(1)

The tangents at P and Q meet at the point N.

Given $p + q \neq 0$,

(c) show that point N has coordinates $\left(\frac{10pq}{p+q}, \frac{10}{p+q}\right)$.

(4)

The line joining N to the origin is perpendicular to the line PQ.

(d) Find the value of p^2q^2.

(5)

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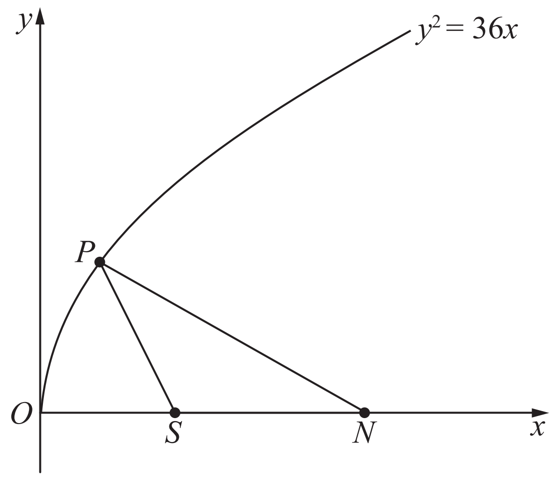

Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 36x$.

The point P (4, 12) lies on the parabola.

(a) Find an equation for the normal to the parabola at P.

(5)

This normal meets the x-axis at the point N and S is the focus of the parabola, as shown in Figure 1.

(b) Find the area of triangle PSN.

(4)

Leave blank

4. The rectangular hyperbola H has Cartesian equation $xy = 4$

The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$

(a) Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

(5)

The normal to H at the point where $t = -\frac{1}{2}$ meets H again at the point Q.

(b) Find the coordinates of the point Q.

(4)

Leave blank

6. A parabola C has equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C, where $p \neq 0$, $q \neq 0$, $p \neq q$.

(a) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at P meets the tangent at Q at the point R.

(c) Find, in terms of p and q, the coordinates of R, giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C,

(d) find the value of pq.

(2)

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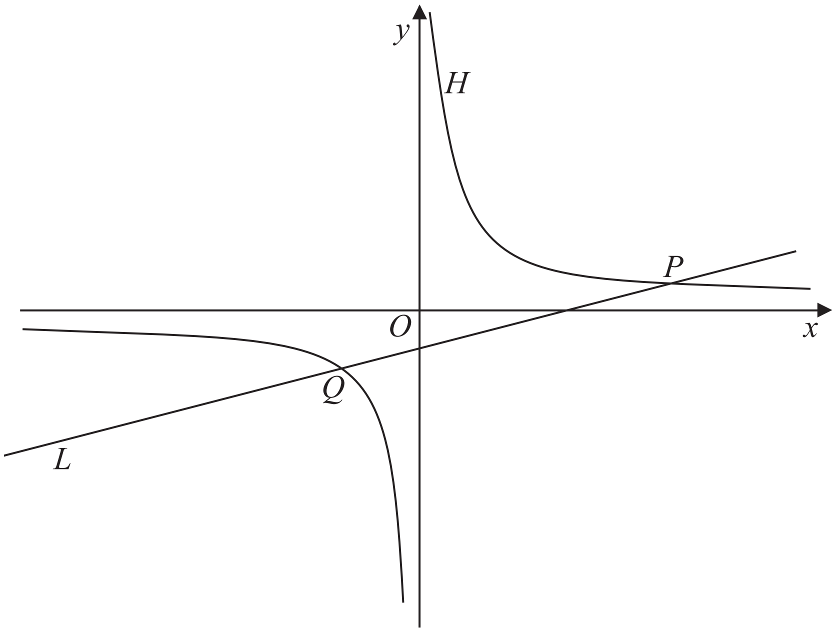

Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q.

(5)

Leave blank

7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C.

(a) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y-axis at a point Q.

(b) Find the coordinates of Q.

(1)

Given that the point S is the focus of C,

(c) show that PQ is perpendicular to SQ.

(3)

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$